Composition of Self Organizing Maps for Adaptive Mesh Construction on Complex-shaped Domains

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Abstract— In this paper, an important application of Self-Organizing Maps (SOM) to construction of adaptive meshes is considered. It is shown that application of the basic SOM model leads to a number of problems like inaccurate fitting the border of a physical domain, mesh self-crossings, etc. The composite SOM model is proposed which is based on the composition of a number of SOM models interacting in a special way and self-organizing over their own set of input data. A core of the composite SOM model is the colored SOM model with nonadjustable neurons which provides us a technique to control the neuron weights adjustment taking into account the fixed ones and the general layout of the mesh. As a result, the composite SOM model allows us to approximate an arbitrary complex physical domains with well topology preservation.

1 Introduction

Self Organizing Map (SOM) is a neural network that has been used in a wide range of scientific and industrial applications [1]. The ability of SOM model to perform the topology preserving mapping of high dimensional data onto a low dimensional space makes it possible to apply it to the construction of adaptive meshes used in the area of complex numerical simulation problems [2].

Within the scope of all types of adaptive meshes, there is an important class in which the mesh is an image under an appropriate mapping of a fixed mesh. All conventional methods of this class, such as equidistribution method [3], Thompson method [4], elliptic method [5], etc., and even algebraic and conformal mapping ones, eventually require solving a complicated system of nonlinear partial differential equations (PDEs) to obtain good enough adaptive meshes. The necessity of solving PDEs usually leads to significant difficulties, among which are those connected with initial mesh, limitations on mesh density function, efficient parallelization, etc. [6]. Additionally, special complex PDEs for mesh construction are required for different dimensionalities of a physical domain.

Self-organizing properties, inherent parallelism and stochastic nature of the SOM learning algorithm form an essential basis for the development of highly efficient methods of mesh construction [6]. Unlike the conventional methods, the SOM is expected to allow us to construct adaptive meshes with arbitrary initial data, without limitations on the mesh density function. Moreover, it is possible to make the process of mesh construction fully automatic, particularly, there is no need to fix the boundary nodes beforehand, and the algorithm of mesh construction can be made universal with regard to dimensionalities of the physical domain [7].

When applying a basic SOM [1] for the mesh construction, there are a number of problems. First, it is impossible to obtain an accurate approximation of the border of a physical domain; and second, the failures of topology preservation can lead to mesh self-crossings and result in the mesh nodes going out of the physical domain when constructing the mesh over the non convex domains and with complicated mesh density distribution. These problems make it difficult to obtain qualitative adaptive meshes [8].

In this paper, the composite SOM model is proposed. This model is based on the composition of a number of SOM models interacting in a special way and self-organizing over their own set of input data. The SOM models taking part in the composition are responsible typically for the border or interior of a physical domain. The main task of the learning algorithm for the composite SOM model is to provide the consistency between these SOM models. The composite SOM model allows us to overcome the problems of the basic SOM listed above and, thus, to construct qualitative meshes automatically over complex, even multiply-connected, physical domains.

There are several works in which an attempt of joining together a number of SOM models, self-organizing over given sets of input data, has been made.

The first has been developed for 3D shape reconstruction for mobile robotics [9]. Input data is divided into subsets by clustering and then each subset of data is used for training of a corresponding SOM model.

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After the training, some of the SOMs are joined by connecting their nearest boundary neurons. The approach is not suitable for mesh construction because joining is performed without taking into account the topology of SOM models and is based only on the closeness of neuron weights. In the proposed composite SOM model, joining is accomplished automatically during the learning.

For adaptive meshes, in [10] the interweaving algorithm has been proposed which brings together 1D SOM for the boundary nodes, and 2D SOM for the interior ones. But this algorithm interferes in the input data occurrence that can lead to the distortion of mesh density function. The more developed composite algorithm is proposed in [8]. The composite algorithm is based on the special alternation of 1D and 2D basic SOM models. But it still does not allow the mesh to fit in an appropriate way the essentially non-convex domains. The composite algorithm serves as a background for the learning algorithm of the proposed composite SOM model.

A core of the composite SOM model is a SOM-like model that may involve a number of nonadjustable neurons with a technique balancing the border effect for the small learning radius and coloring the neurons and input data. Using appropriately these neurons together with the coloring technique, it is possible to essentially improve the topology preservation. The coloring technique is close to the multi-block adaptive meshes approach used in conventional PDE-based methods [4].

On the one hand, the use of nonadjustable neurons and coloring limits freedom of self organization of SOM. But on the other hand, from a practical point of view, the composite SOM model becomes more flexible and capable of accurate approximation of input data with any distribution.

We believe that the composite SOM model is applicable not only in the field of adaptive mesh construction but in other areas in which it is possible to divide input data into subsets or separate border and interior data.

The paper is organized as follows. In Section 2, the application of the basic SOM model is discussed, the learning rate suitable for adaptive mesh construction is presented. Section 3 contains the description of the colored SOM model with nonadjustable neurons is proposed. In Section 3, the composite SOM model, its architecture and the learning algorithms are proposed. Section 5 concludes the paper.

2 The basic SOM for adaptive mesh construction

Let the SOM neuron layer consists of N neurons. Each i neuron has a fixed location q_i in the Euclidean space R_Q, where q_i is a point in the given computational domain Q. Therefore, the neuron layer forms a mesh $Q_N = \{q_1, ..., q_N\}$ over Q. Let a map M be referred to the set of neuron indices $\{1, ..., N\}$. For simplicity, it is assumed that the mesh Q_N is a rectangular uniform one, but all the techniques proposed in this work can be applied for meshes of other structures. For each pair of neurons, i-th and j-th, $i, j = 1, ..., N$, there is a lateral connection between them with strength being a decreasing function of the distance between q_i and q_j.

Let G be a physical domain, in the Euclidean space R_G, on which an adaptive mesh $G_N = \{x_1, ..., x_N\}$ is to be constructed, where $x_i \in G$, $i = 1, ..., N$ are desired adaptive mesh nodes locations. When applying SOM for adaptive mesh construction, the point x_i is a weight vector of the corresponding i-th neuron which is updated during the learning process. Random points from G serve as the input data for SOM. Density distribution of the resulting mesh is controlled by the probability distribution used for random point generation [8].

The learning algorithm for the basic SOM model consists of the following steps. At each iteration t, a random point y is generated from G; among all the neurons the winner is selected, which has the weight vector $x_m(t)$ being closest to the y; and all the neurons adjust their weights according to the following rule:

$$x_i(t+1) = x_i(t) + \delta(t)\eta_{q_m}(t, q_i)(y - x_i(t)) \,. \qquad (1)$$

where $\delta(t) \in [0,1]$ is responsible for a learning step and $\eta_{q_m}(t, q_i) \in [0,1]$ is a function which defines the strength of the lateral connection between the m-th and i-th neurons. These two functions control the magnitude of nodes displacements in G while the nodes move towards the point y, and essentially influence on the quality of resulting meshes and speed of construction process. The composite SOM model employ the same learning rule (1) as the basic SOM model, but this rule is applied in specific conditions as it is described in the next Sections.

The learning rate selection is very important issue when applying the SOM for mesh construction. Therefore, based on the experiments the learning rate has been thoroughly selected to provide the good mesh quality with reasonable computational speed. The learning step is define by the function $\delta(t) = t^{-0.2}\chi(t)$, where $\chi(t) = 1 - e^{5(t-T)/T}$ and T is a maximum number of iterations which is fixed beforehand depending on N. In our experiments, $T = 10N$. The function for lateral connections has the following form: $\eta_{q_m}(t, q_i) = s^{\left(\frac{d(q_m, q_i)}{r(t)}\right)^2}$, where $s \in (0,1)$ is fixed to be close to zero, e.g. $s = 10^{-5}$, and $r(t)$ is a learning radius which is a decreasing function of t and given by $r(t) = r(T) + \chi(t)\left(r(1)0.05^{t/T} - r(T)\right)t^{-0,25}$. Here $r(1)$ and $r(T)$ initial and final radiuses, $r(1) > r(T)$. The learning rate provide the condition that the winner

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receives the maximum displacement, while for the other nodes the greater the distance between them, the less their weights change.

When applying the basic SOM for mesh construction, the following problems occur. First, it is impossible to obtain accurate approximation of the border of a physical domain, as it can be clearly seen from all of the examples in Fig. 1, because boundary nodes never reach the border and they are influenced by the border effect. Second, some of the mesh nodes can go out of the domain in the case of complex non-convex domains (Fig. 1(a)). Third, a general layout of the mesh is fixed unpredictable and can turn out to be unsuitable for the given configuration of the domain (Fig. 1(a)). Fourth, if the probability distribution p(x) is non uniform, then boundary nodes can propagate to the interior of the domain that is the result of bad topology preservation as Fig. 1(b) shows. Finally, the mesh may contain self-crossings (Fig. 1(c)) that makes it entirely unusable for numerical simulations. All of these problems can be solved by using the composite SOM model presented below.

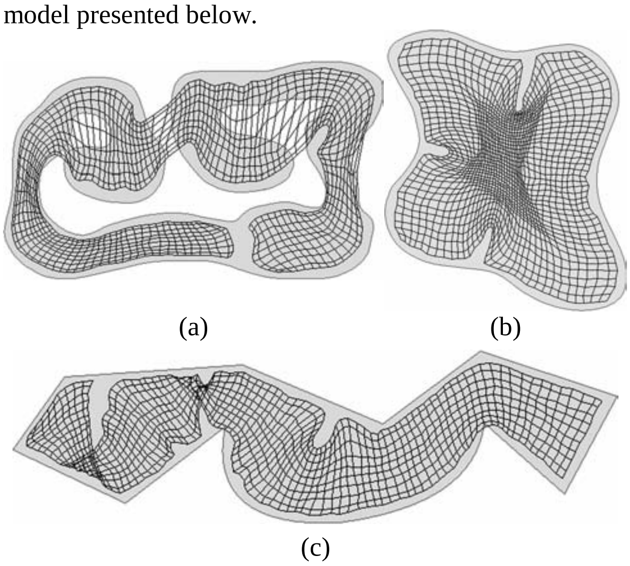

Fig. 1. Problems of the basic SOM application to adaptive mesh construction. (a) mesh nodes go outside the non convex domain and the mesh layout is unsuitable; (b) boundary nodes propagate to the interior of the domain; (c) mesh self-crossings.

3 Colored SOM with non-adjustable neurons

A core of the composite SOM model is a SOM-like model that involves a number of non adjustable neurons and the technique of coloring the neurons and input data.

Let $F \subseteq M$ be a subset nonadjustable neurons. The weights of these neurons are fixed in some way and not adjusted during the learning process. The task is how to organize the learning process for adjustable neurons updating their weights in consistency with the nonadjustable ones. Due to the lateral connections, according to the learning rule (1), neurons change their weights in the same direction as the winner, with the magnitude depending on the distance to the winner in Q. Therefore, the nonadjustable neurons should participate in the winner selection process.

Nonadjustable neurons don't change their weights towards the random point y while the other neurons do. To provide consistency, the point y is to be replaced by the location of the mesh node x_m if the m-th neuron is the winner and nonadjustable one. As a result, the mesh nodes move directly towards a fixed node once it becomes a winner. It has to be noted that replacing the random point by the nonadjustable winner balances the border effect if the final learning radius $r(T)$ is small.

If the weights of nonadjustable neurons are fixed appropriately, the topology preservation can be improved essentially, e.g. it is possible to avoid mesh self-crossings and to exclude the situation when boundary nodes propagate to the interior of the physical domain. Also, this technique allows us to obtain the mesh without nodes outside the physical domain, even in the case of complex non-convex domain like the one shown in Fig. 1(a).

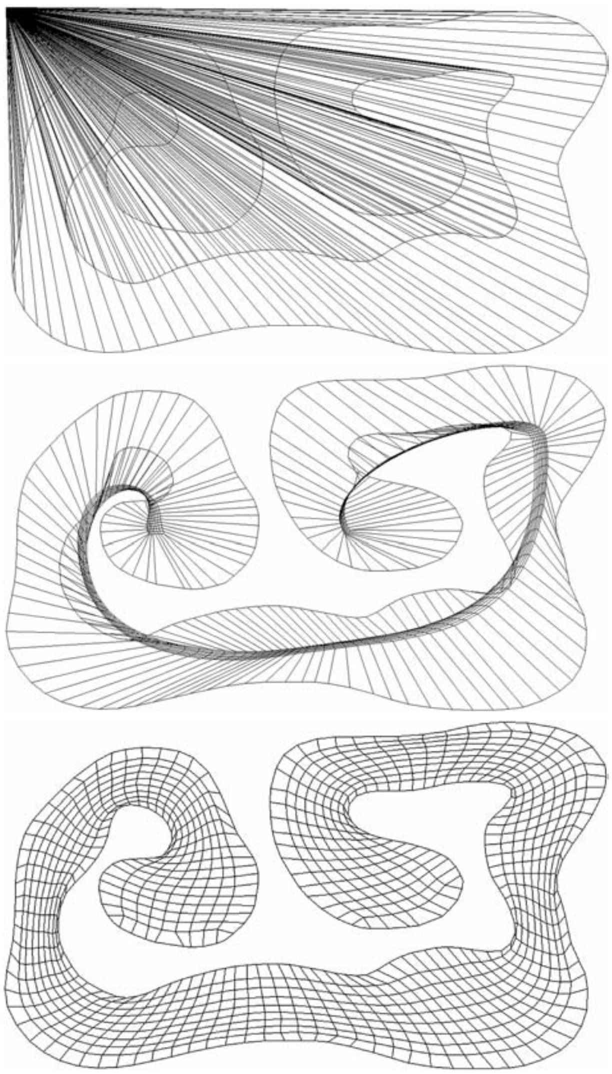

Fig. 2. The process of mesh construction with nonadjustable boundary neurons: 1st, 10th and 10000th iterations.

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In this example, all the boundary mesh nodes are placed to their correct positions along the border of G and has been declared as nonadjustable. Initial locations of the interior mesh nodes has been set up to (0,0). It can be seen that the fixed boundary nodes induce the correct mesh lay-out (Fig. 2) and the interior nodes finally fit the boundary ones in an appropriate way. Such a distribution of mesh nodes is unreachable when using the basic SOM (compare with Fig. 1(a)).

The next question is how to obtain appropriate weights for nonadjustable neurons? For example, it is still a non-trivial task to fix the boundary nodes along the border beforehand like it's done in Fig. 2. As a solution to this question, we propose the composite SOM model which is able to alternate construction of the mesh over particular parts of the domain and, thus, to distribute all mesh nodes over the domain automatically based on the self-organization. Nonadjustable neurons then serve as the basis for providing the consistency between those different parts of the mesh. For example, at the stage of updating the interior nodes, the boundary ones can be considered as nonadjustable, while the correction of boundary nodes is performed with fixed interior nodes.

Since the composite SOM model distribute the mesh nodes based on self-organization, there is a chance to obtain incorrect mesh layout. To make the composite SOM model flexible, let us introduce the coloring technique which helps the model to detect a correct mesh layout. Although this model is capable to detect the layout by itself, the use of the coloring technique makes it possible to construct a mesh over an arbitrary complex physical domain by dividing it into a number of more simple ones.

Let $C_G : G \to \{c_1,...,c_p\}$ be a coloring function which puts each point of G into the correspondence with one of the colors $c_1,...,c_p$. Similarly, a coloring function $C_Q : M \to \{c_1,...,c_p\}$ defines the colors for neurons. These functions take part in the learning algorithm in such a way that at each iteration the winner is selected from neurons of the same color as the random point y. As a result, mesh nodes of the color c_j can become winners only when y is generated from the subdomain of the same color c_j, and then, they gradually move towards this subdomain. In Fig. 2, boundary nodes has been distributed using the coloring functions shown in Fig. 4. The aim of coloring in this case is to separate parts of the domain border which are close to each other.

Before presenting the composite SOM model, let us list the learning algorithm for colored SOM model with nonadjustable neurons (the procedure SOM-Core) which is used at each alternation stage of the learning algorithm for the composite SOM model and processes a part of the mesh.

Algorithm 1. The procedure SOM-Core.

Repeat the following operations at each iteration $t = t_{st},...,t_{fin}$:

(1) Generate a random point $y \in G$ according to the probability distribution $p(x)$.

(2) Calculate the Euclidean distances $d(\cdot,\cdot)$ between y and all the weights $x_i(t)$ for which $C_G(y) = C_Q(i)$, and choose the winning neuron with weights $x_m(t)$, where

$$m = \arg\min_{i \in M} \{ d(y, x_i(t)) \mid C_G(y) = C_Q(i) \}.$$

(3) If $m \in F$, i.e. the winner is nonadjustable, then the random point y is to be replaced by the weight vector of the winner: $y := x_m(t)$.

(4) Adjust weights of all neurons with indices from $M \setminus F$ using the following rule:

$$x_i(t+1) = x_i(t) + \delta(t)\eta_{q_m}(t, q_i)(y - x_i(t)),$$

where $i \in M \setminus F$.

4 Composite SOM model

The idea of the composite SOM model is to combine together a number of SOM models interacting between each other in a special way and self-organizing over their own set of input data. Learning algorithm for the composite SOM model is based on the alternation of training of each SOM model by the *Algorithm 1*.

Let the physical domain G be divided into subdomains $G_1,...,G_n$. The neuron layer is also divided into n subsets. Each k-th subset of neurons forms a part of the mesh which is to be spread over the subdomain G_k. A simple example of such a division is when we separate the border and interior of a physical domain and divide all mesh nodes into boundary and interior nodes. This kind of division seems to be the most convenient for the majority of physical domains.

To specify a SOM model for each k-th part of the mesh, it is necessary to define a map $M_k \subseteq M$, $k = 1,...,n$. Since each k-th part is to be adapted to a number of neighboring parts of the mesh, the map M_k should contain not only neurons from the k-th part but the ones from the neighboring parts of the mesh which are the nearest neighbors for the neurons of the k-th part. These neighboring neurons are considered as nonadjustable while training the k-th SOM model. But they can adjust their weights during another alteration stages. The set of all nonadjustable neurons for the k-th SOM model is denoted by F_k, where $F_k \subseteq M_k$, $k = 1,...,n$. An example of the collection of maps M_k is shown in Fig. 3. In this figure, two 1D meshes and one 2D mesh are to be consistently constructed over a multiply-connected physical domain.

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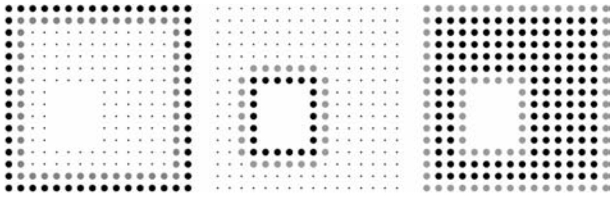
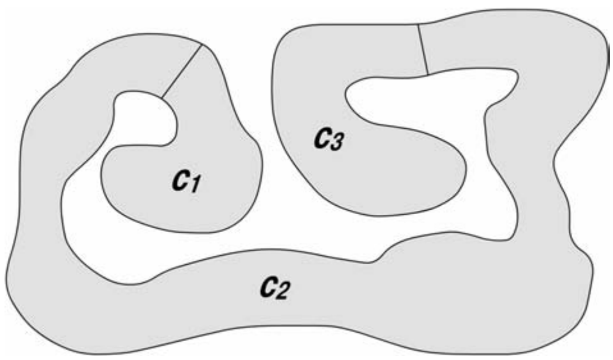


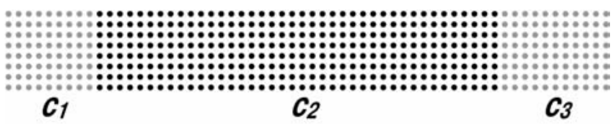
Fig. 3. Collection of 3 submaps for construction of the mesh over a multiply-connected physical domain. Black neurons are adjustable and gray are nonadjustable.

Furthermore, the coloring functions C_G and C_Q are given which are defined over the whole physical domain G and the map M respectively. An example of the coloring functions is shown in Fig. 4. These function has been used for construction the boundary mesh in Fig. 2.

A collection of maps M_k together with the sets of nonadjustable neurons and the coloring functions constitutes the architecture of the composite SOM model. This architecture depends on the configuration of the physical domain.



(a)



(b)

Fig. 4. Coloring functions. (a) the function C_G; (b) the function C_Q. These functions has been used for construction of the boundary mesh in Fig. 2.

Each alternation stage of the learning algorithm for the composite SOM model consists in training of all the SOM models during a given number of iterations, is referred to as a macroiteration and is denoted by s. For each map M_k, there is a private counter of iterations, and the maximum number of iterations T_k is given in such a way that T_k is proportional to $|M_k|$, i.e. to the number of neurons in the map M_k. Let $\varphi_k(s)$ be the number of iterations at the macroiteration s during which the procedure SOM-Core is to be applied to the k-th SOM model. For example, $\varphi_k(s) = T_k / S$, where S is the maximum number of macroiterations. The functions $\varphi_k(s)$ can be chosen depending on the physical domain configuration. The learning algorithm for the composite SOM model consists of the following steps.

Algorithm 2. Learning algorithm for the composite SOM model.

(0) Set arbitrary initial weights of all neurons $x_i(0)$, $i = 1,...,N$.

(1) At the first macroiteration ($s = 1$), apply the procedure SOM-Core to the general map M without nonadjustable neurons, i.e. $F = \varnothing$, with random points generated from the whole domain G and $t_{st}(1) = 1$, $t_{fin}(1) = T_0$, where T_0 is a given number of iterations.

(2) Repeat the following operations at each macroiteration $s > 1$: for each $k = 1,...,n$ apply the procedure SOM-Core to the map M_k with nonadjustable neurons F_k, random points generated from G_k and $t_{st}(s) = t_{fin}(s-1)+1$, $t_{fin}(s) = t_{st}(s) + \varphi_k(s)$.

The step (1) of the Algorithm 2 is a ordering stage of the learning algorithm. Application of SOM-Core to all mesh nodes makes the mesh become ordered and take roughly the form of G. The number of iterations T_0 depends on the physical domain configuration. Typically, T_0 is varying from 0.01T to 0.005T. Due to the coloring functions, the correct mesh layout is reached after this step, and boundary nodes are located near their appropriate border positions.

The step (2) is a refining stage of the learning algorithm. All the submaps consistently fit more and more fine details of their own part of the physical domain. Overlappings between the submaps help them to keep in touch with each other, and the alternation controls all the weights to change gradually. From our experiments, in some cases the better results can be obtained if the submap for outer boundary neurons does not have nonadjustable interior neurons. Actually, it means that the outer boundary nodes is responsible for the topology and all other nodes adapt to them.

In Fig. 5, examples of adaptive meshes constructed using the proposed composite approach are shown. Quality of 2D meshes in Fig.5 has been measured by the generally accepted quality criteria for quadrilateral meshes such as the criteria of cell convexity and oblongness, the criterion of mesh lines orthogonality [8]. The values of these criteria are in the admissible range. The adaptive mesh over a multiply connected domain, shown in the center of Fig. 5, has been constructed for test numerical simulations of a solitary wave run-up around islands where each island assigns a hole in a physical domain [3]. Here mesh density is defined by ocean depth values.

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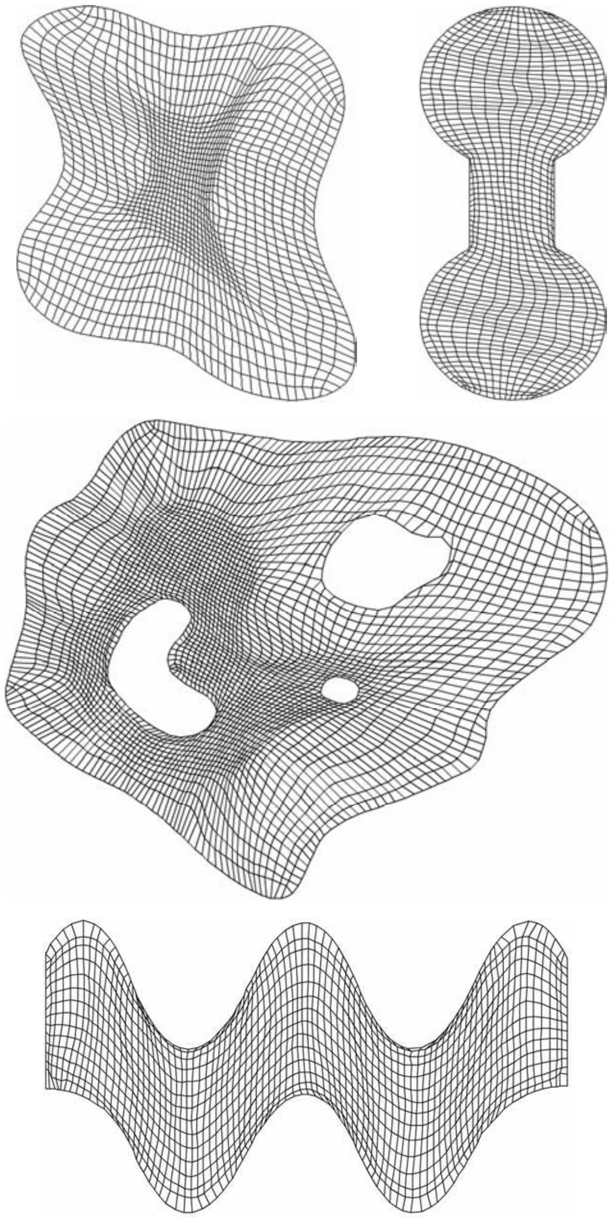

Fig. 5. Examples of adaptive meshes constructed by the proposed approach of the composition of Self Organizing Maps.

5 Conclusions

The proposed method based on the composite SOM model provides us an efficient and automatic tool for adaptive mesh construction without limitations on an initial mesh and mesh density function and does not require to fix mesh nodes along the border beforehand, since the proper distribution of boundary nodes is detected during the learning process. At the same time, the quality of the resulting meshes constructed by the proposed composite approach has been evaluated as acceptable according to the commonly used quality criteria for finite-difference meshes. One of the most important feature of the proposed method is that it can be easily parallelized with efficiency greater than 90% [6].

In the future, the composite model that combines a number of Growing Neural Gas models [11] is to be developed. This model is expected to provide us no less efficient method of unstructured adaptive mesh generation. Also, the SOM with nonadjustable neurons is to be applied for generation of moving adaptive meshes, since it can provide us a technique to control local adaptive mesh refinements without global mesh reordering.

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